A Distributed Localization Algorithm for Wireless Sensor Networks Based on the Solutions of Spatially-Constrained Local Problems

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***Abstract*— We present a distributed localization algorithm for wireless sensor networks. Each sensor estimates its position by iteratively solving a set of local spatially-constrained programs. The constraints allow sensors to update their positions simultaneously and collaboratively using range and position estimates to those neighbors within their communications range. Moreover, the algorithm is designed for implementation with resource-limited devices. Since the exchange of information among sensors is a key component for this method, we introduce a stopping criterion to monitor the wireless transmissions for the whole network in order to significantly reduce energy consumption with minimal impact on localization accuracy. Experimental results show that we can determine the best tradeoff between wireless transmissions and accuracy. The performance of the proposed scheme is very competitive when compared with similar and more computationally demanding schemes.**

***Index Terms*— Distributed-localization, wireless sensor networks.**

NOMENCLATURE
ACRONYMS AND SYMBOLS

Acronym	Description
AoA	Angle of Arrival
BIL	Bilateration
dB	Decibels
DSCL	Distributed Spatially Constrained Localization-Local
dwMDs	Distributed Weighted Multidimensional Scaling
GOF	Global Optimization Function
GPS	Global Positioning System
LM	Levenberg–Marquardt
LS	Least Squares
ML	Maximum Likelihood
NLLS	Non Linear Least Squares
PPE	Push-Pull Estimator
QP	Quadratic Programming
RF	Radio Frequency
RMSE	Root Mean Square Error
RSS	Receive Signal Strength
RSSI	Receive Signal Strength Indicator
SD	Standard Deviation
SDP	Semidefinite Programming
SQP	Sequential Quadratic Programming
TDoA	Time Difference of Arrival
ToA	Time of Arrival
ToF	Time of Flight
WLS	Weighted Least Squares
WSN	Wireless Sensor Network
WSNs	Wireless Sensor Networks

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SYMBOLS

Symbol	Description
$\mathbf{S}$	Set of sensors
N	Number of sensors in the network
$\mathbf{S}_i$	Set of neighboring sensors of the sensor s_i
N_i	Number of elements on set $\mathbf{S}_i$
$\mathbf{p}_i$	Position estimate for the sensor s_i
$\mathbf{z}_i$	True location for the sensor s_i
$\mathbf{A}$	Set of M anchors
M	Number of anchors in the network
$\mathbf{A}_i$	Set of neighboring anchors of the sensor s_i
$\mathbf{q}_i$	True location for the anchor a_i
R_{ik}	Range estimate between the anchor a_k and the sensor s_i
d_{ik}	True distance between the anchor a_k and the sensor s_i
r_{ij}	Range estimate between two sensors s_i and s_j
$\mathbf{L}$	Set of position estimates
$\mathbf{p}_i^0$	Initial estimates of the sensor s_i
$\mathbf{p}_i^\ell$	Position estimate of the sensor s_i at the iteration ℓ
α_{ij}^ℓ	Range error between the sensor s_i and the sensor s_j at the iteration ℓ

β_{ik}^{ℓ}	Range error between the sensor s_i and the anchor a_k at the iteration ℓ
Ω_i	Constrained search region centered at the location $\mathbf{p}_i^{\ell}$
η_p	Path-loss exponent
δ_i	Parameter used to constraint the search region
$\overline{P}_{ij}$	Expected power measurement received by the sensor s_i from the sensor s_j
P^{ij}	Measured power between a sensor s_i and a sensor s_j
$\mathrm{C}(\cdot)$	Global cost function
$\|\cdot\|$	Norm 2
min	Minimize
σ_{SH}	Noise in decibels
ε	Threshold for the stopping criterion $\left\|\mathbf{p}_i^{\ell} - \mathbf{p}_i^{\ell+1}\right\|$

I. Introduction

WIRELESS sensor networks (WSNs) are making inroads into the most varied applications. They offer the ability to acquire information at spatial and time scales which were difficult, expensive, or impossible to achieve previously. The low cost of the sensors and the savings on infrastructure allows deployments of tens, hundreds, or even more devices equipped for specific applications. WSNs are a technological breakthrough that is changing the social landscape as they are integrated into different aspects of our lives like health care, homeland security, infrastructure monitoring, and transportation to name a few.

On a wireless sensor network (WSN), it is a common assumption to deploy sensors over a region with limited to non-existent control on their position in space. This situation has given rise to a significant amount of work on self-localization schemes for WSNs [1]–[4]. The problem of WSN localization is preceded by work on statistics and target-source localization for military and communication applications [5]–[7]. Localization has been an active area of research in WSNs from the beginning. Knowledge of the sensor positions is crucial to establish the network topology, track objects, monitor an event, determine the quality of coverage, move data through the network, and to determine spatial/geographical relationships for data mining and signal analysis [8].

The simplest way to determine location on WSNs consists of integrating a global positioning system (GPS) in each sensor. This option has the advantage of geographical precision, but increases the cost, energy-consumption, and the size of a sensor (three parameters that always should be minimized in a WSN). Also, GPS requires line-of-sight between the sensor and the satellites [9] which is not possible on indoor environments. A more efficient scheme from the *cost-energy-precision* point of view consists of equipping only a small fraction of the sensors with GPS capabilities. These sensors serve as anchors or reference sensors which can be used by the remaining sensors to estimate their own positions using range estimates (i.e., distances) to them. Methods to estimate the range among sensors include the time of arrival (ToA), time difference of arrival (TDoA), received signal strength (RSS), or angle of arrival (AoA) [10]–[14].

Centralized schemes [1], [8] solve the localization problem in a base station or processing center, where all pair-wise distances between sensors are gathered via wireless transmissions. Thus, energy-conservation and robustness of centralized schemes are affected by the network size, topology, and sensor range. On the other hand, if the mathematical model for the WSN localization problem is solved in a distributed form (i.e., each sensor being able to estimate its own position), the amount of wireless communications among sensors could be greatly reduced. Moreover, the whole WSN can be tolerant to node failures [15].

Recently, many distributed algorithms have been proposed coming from different perspectives. In [16], a novel distributed weighted multidimensional scaling (dwMDS) that corresponds to the weighted least squares (WLS) approach is proposed. This algorithm is a variation of classical centralized MDS [17]. In a similar way, [18] using the non-linear least Squares (NLLS) approach divides a global optimization function, subject to non linear geometrical constraints, into local optimization functions that are solved in a distributed way through the Gauss-Newton method. Other localization algorithms have formulated the WSN localization problem like a semidefinite programming (SDP) problem where interior point methods are successfully applied [19]–[21]. Using a different approach, a push-pull physical model is applied in [22] to design a distributed localization algorithm, where force vectors are used to iteratively re-estimate the positions of each unknown sensor until convergence is achieved. Also a grid search non-iterative method can be used as shown in [23]. For each point in the grid a function is calculated and the correct extreme is found.

In this paper, we consider modeling the WSN localization problem as a non-convex problem with non-linear constraints where a non-linear optimization algorithm should be used to obtain a solution [24]. We propose a distributed iterative localization algorithm which aims to achieve good accuracy within few iterations. At the end of each iteration all sensors update and broadcast their positions to neighboring nodes. During the update phase, each sensor solves a spatially-constrained program using range and position information from local neighbors. We introduce a local objective function where a sensor attempts to minimize the mean absolute range error with all its neighbors. The algorithm is characterized by a spatial constraint that limits the solution space to a region around the current position estimate. We note that the proposed approach has computational characteristics that allow its deployment on real sensor hardware. We performed an extensive evaluation of the trade-off between localization accuracy and wireless transmissions. This is a key metric for an iterative scheme as wireless transmissions are the most energy consuming operations for a sensor [25]. We show that for large-scale network scenarios, it is possible to determine a good trade-off point between localization accuracy and the amount of required wireless transmissions.

This paper is organized as follows. Section II presents a review of well known ranging techniques which are used in our localization algorithm. In Section III we formulate the range-based localization problem within the context of non-linear programming. In Section IV we introduce a distributed localization algorithm with spatial constraints that allow collaboration among sensors during the localization process. Section V evaluates and compares the accuracy performance and wireless transmissions of our proposed algorithm with two well-known iterative schemes. Finally, we discuss conclusions in Section VI.

II. Range Estimation Techniques

Range-based techniques estimate the true distance between two sensors using time-of-flight (ToF), power, and/or angle measurements. ToF measures the time that it takes for a signal (e.g., acoustic, radio frequency (RF), or other) to travel from a sensor s_i to a sensor s_j, and it basically presents two modalities: the ToA and $TDoA$ schemes [13], [14]. These techniques require additional sensor hardware to detect signals and make accurate timing measurements. Knowing the ToF from a sensor s_i to a sensor s_j and the velocity-propagation of the signal v_p, the distance between the two sensors can be primarily formulated as $d_{ij} = v_p \cdot ToF$. However, this first approximation does not take into account environmental factors like additive noise, shadowing, multi-path signals, and internal time delays (τ_D) on the sensors, so more sophisticated models which take into account more specific information must be developed as shown in [26]–[29]. In synchronous sensor networks the τ_D problem can be easily solved, but in asynchronous sensor networks two-way ToA measurements are commonly used in practice [30].

The RSS method is based on the measurement of the signal power. This technique is popular since RSS hardware is commonplace in wireless devices and can be used to estimate the distance between two sensors [31]. As a first approximation, considering the free space path loss model, the distance d_{ij} between two sensors s_i and s_j can be estimated by assuming that the power signal decreases in a way that is inversely proportional to the square of the distance $(1/d_{ij}^2)$. However, in real environments the signal power is affected by a factor $(1/d_{ij}^{\eta_p})$. The path-loss factor η_p is closely related to geometrical and environmental factors (e.g., shadowing, reflection, diffraction, scattering, and fading), and it varies from 2 to 4 for practical situations [28].

In this paper, we use the log-distance path loss model denoted as

$$P^{ij} = P_0(d_0) - 10\eta_p \log_{10}\left(\frac{d_{ij}}{d_0}\right) + \sigma_{\mathrm{SH}} = \overline{P}^{ij} + X_\sigma \quad (1)$$

where P^{ij} is the power path loss (measured in dB) between the two sensors, $P_0(d_0)$ represents the measured power at the reference distance d_0 from the transmitter (typically $d_0 = 1$ meter), and X_σ is a zero-mean Gaussian random variable with standard deviation σ_{SH} for the case of shadowing. Hence, the noisy power measurements follow the distribution

$$P^{ij} \sim \mathcal{N}(\overline{P}^{ij}, \sigma_{\mathrm{SH}}) \quad (2)$$

in decibels which correspond to a log-normal distribution of the power loss in Watts. From (1) and (2) the distance estimate r_{ij} between a sensor s_i and a sensor s_j can be obtained as follows:

$$r_{ij} = d_0 \cdot 10^{\frac{P_0(d_0) - P^{ij}}{10 \cdot \eta_p}}. \quad (3)$$

III. Problem Description

Consider a set of N sensors $\mathbf{S} = \{s_1, s_2, \ldots, s_N\}$, randomly deployed over a region. For a two-dimensional scenario (i.e., 2-D) denote the *true* position for sensor s_i as $\mathbf{z}_i = [z_{x_i}, z_{y_i}]^T$ which $\mathbf{z}_i$ is unknown and its estimated position as $\mathbf{p}_i = [p_{x_i}, p_{y_i}]^T$. Furthermore, assume the random deployment of M anchors $\mathbf{A} = \{a_1, a_2, \ldots, a_M\}$ which are equipped with GPS or any other scheme to self-localize. The location of an anchor a_k is represented by $\mathbf{q}_k = [q_{x_k}, q_{y_k}]^T$. Also, in practice it is common to have $M \ll N$ with $M > 2$.

In a range-based localization scheme it is assumed that each sensor can estimate the distance to other sensors using ToA or RSS. Thus, the range estimate between s_i and s_j can be denoted as

$$r_{ij} = r_{ji} = d_{ij} + e_{ij} \quad (4)$$

where $d_{ij} = \|\mathbf{z}_i - \mathbf{z}_j\|$, $\|\cdot\|$ is the Euclidean distance, and e_{ij} represents the range error introduced by environmental noise, propagation distortion, and the ranging technique. In a similar way, the range estimate between a sensor s_i and an anchor a_k can be defined as

$$R_{ik} = R_{ki} = d_{ik} + e_{ik} \quad (5)$$

where $d_{ik} = \|\mathbf{z}_i - \mathbf{q}_k\|$. Moreover, assume that all sensors have a limited number of neighboring sensors constrained by the range of coverage γ in the vicinity of a sensor. Thus

$$\mathbf{S}_i = \{j \mid \|\mathbf{z}_i - \mathbf{z}_j\| < \gamma\} \quad (6)$$

defines the known pair-wise distances between a sensor s_i and sensors s_j. Similarly,

$$\mathbf{A}_i = \{k \mid \|\mathbf{z}_i - \mathbf{q}_k\| < \rho\} \quad (7)$$

defines the set of known pair-wise distances between a sensor s_i and anchors a_k also limited by the range of coverage ρ.

Then the problem in WSN localization consists of estimating the position $\mathbf{p}_i$ for each sensor s_i such that the norms of these positions minimize the residuals with the corresponding ranges r_{ij} and R_{ik}. The solution to this problem is one of the most challenging problems in WSNs. The problem can be mathematically formulated as the following global optimization function (GOF) [32], [33].

$$\min_{\mathbf{L}} \sum_{i \in \mathbf{S}} \left(\sum_{j \in \mathbf{S}_i} \left| \|\mathbf{p}_i - \mathbf{p}_j\| - r_{ij} \right| + \sum_{k \in \mathbf{A}_i} \left| \|\mathbf{p}_i - \mathbf{q}_k\| - R_{ik} \right| \right) \quad (8)$$

where $\mathbf{L} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_N\}$ represents the set of positions to be minimized. Equation (8) describes a nonconvex nonlinear problem which is NP-hard to find a global minimum [18], [24], [32]–[37]. The requirement to have a complete set of range measurements exacerbates the complexity of the problem given the limited energy resources available to each sensor. In the following section we develop an algorithm that circumvents these mathematical and engineering limitations.

IV. Distributed Localization Scheme Based on the Solutions of Spatially-Constrained Local Problems

In this section we present a method derived from (8) that accounts for the constrains found in WSNs, communication and computational resources and exploits the WSN tenet of collaborative in-network processing. The proposed algorithm is iterative, distributed, and computationally simple. The use of optimization or iterative methods to find a solution to problems like (8) has been extensively studied in the literature [21]. The challenge consists of solving the WSN localization problem with a distributed iterative method that can provide accurate position estimates within only a few iterations. Since centralized processing would be a resource intensive operation, it is also crucial to establish an approach to distribute the computational and communications load across the WSN. Hence, given a position estimate $\mathbf{p}_i^\ell$, we require for each sensor s_i to produce a position update $\mathbf{p}_i^{\ell+1}$ using only the positions and range estimates from its neighboring sensors ($\mathbf{S}_i$) and anchors ($\mathbf{A}_i$), respectively. Let us define

$$\alpha_{ij}^\ell(\mathbf{p}_i^\ell) = \left| r_{ij} - \left\| \mathbf{p}_i^\ell - \mathbf{p}_j^\ell \right\| \right| \tag{9}$$

as the range error between the sensor s_i and the sensor s_j for iteration ℓ, and

$$\beta_{ik}^\ell(\mathbf{p}_i^\ell) = \left| R_{ik} - \left\| \mathbf{p}_i^\ell - \mathbf{q}_k \right\| \right| \tag{10}$$

as the range error between the sensor s_i and the anchor a_k for iteration ℓ. Thus, it is easy to see that we can write the global cost function as follows:

$$C\left(\mathbf{p}_1^\ell, \ldots, \mathbf{p}_N^\ell\right) = \sum_{i \in \mathbf{S}} F(\mathbf{p}_i^\ell) \tag{11}$$

where

$$F(\mathbf{p}_i^\ell) = \sum_{j \in \mathbf{S}_i} \alpha_{ij}^\ell(\mathbf{p}_i^\ell) + \sum_{k \in \mathbf{A}_i} \beta_{ik}^\ell(\mathbf{p}_i^\ell) \tag{12}$$

represents the total range error between the sensor s_i and its $\mathbf{S}_i$ neighboring sensors and its $\mathbf{A}_i$ anchors. Trying to find the set $\mathbf{L}^\ell$ that globally minimizes the unconstrained problem (11) is a difficult and sometimes intractable problem. On the other hand, we could separate (12) into a series of local problems where each sensor determines locally a position update $\mathbf{p}_i^{\ell+1}$ inside of a constrained-search region Ω^ℓ. Thus, we can define a local spatially-constrained non-linear programming problem as shown the next equation

$$\min_{\mathbf{p}_i^{\ell+1} \in \Omega^\ell} \left(\sum_{j \in \mathbf{S}_i} \alpha_{ij}^\ell(\mathbf{p}_i^{\ell+1}) + \sum_{k \in \mathbf{A}_i} \beta_{ik}^\ell(\mathbf{p}_\mathbf{i}^{\ell+1}) \right). \tag{13}$$

The idea is to constrain problem (12) such that at each iteration ℓ, the updated position of a sensor s_i, $\mathbf{p}_i^{\ell+1}$, has constrained movements along the correct direction. This would allow all sensors to move "collaboratively" across iterations so that each sensor s_i can progressively adapt to the position updates from the other sensors to gradually reduce the margin of error given by α_{ij}^ℓ and β_{ik}^ℓ.

To illustrate the concept Fig. 1 presents a simple case which describes position estimates and range measurements graphically. In this example, there are three anchors named a_j, a_k, and a_l and two sensors s_i and s_j where the sensor s_i needs to update its position. The range estimate R_{ij} is larger than the Euclidean distance $\left\|\mathbf{p}_i^\ell - \mathbf{q}_j\right\|$, R_{ik} is shorter than $\left\|\mathbf{p}_i^\ell - \mathbf{q}_k\right\|$, R_{il} is shorter than $\left\|\mathbf{p}_i^\ell - \mathbf{q}_l\right\|$, and the range estimate r_{ih} is shorter than $\left\|\mathbf{p}_i^\ell - \mathbf{p}_h^\ell\right\|$. Hence, the sensor s_i must move its current position $\mathbf{p}_i^\ell$ to a new position $\mathbf{p}_i^{\ell+1}$ that minimizes the difference between the measured distances and the Euclidean distances to its neighboring sensors and anchors.

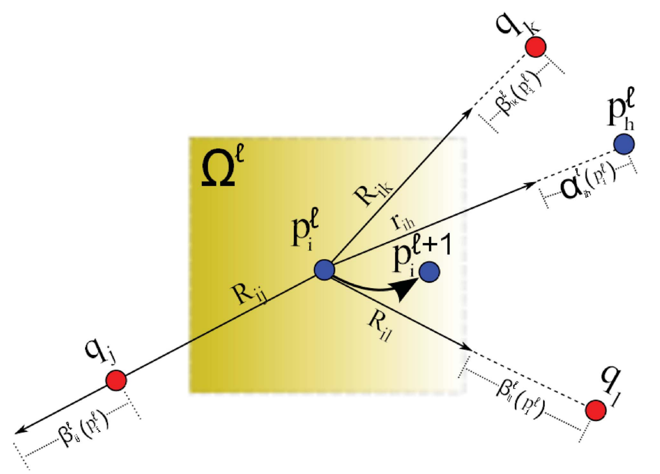


Fig. 1. Sensor s_i updating its position $\mathbf{p}_i^\ell$ using distance measurements and known locations of three anchors (a_j, a_k, a_ℓ) and one neighbor sensor s_h.

Dividing the global function into a set of local minimization problems meets our requirement for an algorithm that is distributed and uses local information from neighbors. Based on (13), we define the constrained search region Ω^ℓ as follows:

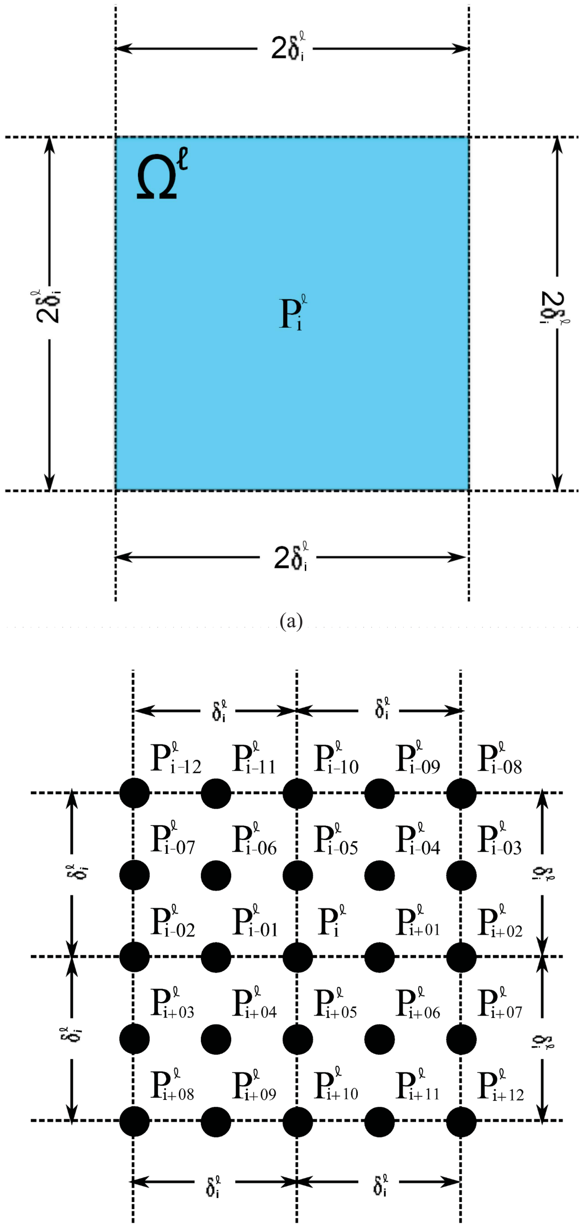
$$\begin{aligned} p_{x_i}^\ell - \delta_i^\ell \le p_{x_i}^{\ell+1} \le p_{x_i}^\ell + \delta_i^\ell \\ p_{y_i}^\ell - \delta_i^\ell \le p_{y_i}^{\ell+1} \le p_{y_i}^\ell + \delta_i^\ell. \end{aligned} \tag{14}$$

Here $p_{x_i}^\ell$ and $p_{y_i}^\ell$ denote the cartesian components of $\mathbf{p}_i^\ell$, and δ_i^ℓ is a heuristic value defined as

$$\delta_i^\ell = \frac{\mathrm{F}(\mathbf{p}_i^\ell)}{N_i} \tag{15}$$

where N_i is the number of elements on set $\mathbf{S}_i$. The constraints $p_{x_i}^\ell$ and $p_{y_i}^\ell$ are geometrical in the sense that they delimit the solution to a 2-D search region. We identify the search region $\mathbf{\Omega}_i^\ell$ as a $2\delta_i^\ell \times 2\delta_i^\ell$ box centered at $\mathbf{p}_i^\ell$ as depicted in Fig. 2(a). We opted to use a square search region (as opposed to circular or some other shape) to achieve lower computational complexity on the actual implementation of the algorithm as discussed next.

The constrained non-linear programming problem described in (13) could be solved with an interior point method [21]. However, the implementation of such methods may be prohibitive for a sensor given the computational and storage limitations of the hardware. We propose the discretization of each Ω_i^ℓ over a 5×5 grid as shown on Fig. 2(b). Only a set of 25 candidate solutions $\mathbf{P}_c^\ell$ for $(-12 \le c \le 12)$ is considered. The value at $c = 0$ corresponds to $\mathbf{p}_i^\ell$. The resolution of 25 candidates of the search region was a trade-off between accuracy and time-computation. Using a lower



(a)

(b)

Fig. 2. Constrained continuous-search and discrete-search areas used iteratively by a sensor s_i to update its actual position $\mathbf{p}_i^\ell$.

resolution grid (e.g., 4 × 4 candidates) in the search region reduced computation requirements of a sensor at the expense of increasing the error on position estimates. On the other hand, increasing the resolution of the search region (e.g., 7×7 candidates) provided better position estimates. However, extensive simulations indicated that the accuracy improvements obtained with 49 candidates were marginal. Hence, using 25 candidates provided the best computation-accuracy trade-off in this work.

The position update $\mathbf{p}_i^{\ell+1}$ can be easily obtained by minimizing (13) over the candidate set using direct substitution. To avoid oscillation in the position estimates, an averaging filter

$$\mathbf{p}_i^{\ell+1} = \frac{\mathbf{p}_i^{\ell+1} + \mathbf{p}_i^{\ell}}{2} \tag{16}$$

is applied using the two most recent position updates.

Algorithm 1 Sensor s_i Refining its Current Position $\mathbf{p}_i^\ell$ Using the DSCL Approach

Require: $\mathbf{p}_i^0$,ε, R_{ik} where $k \in \mathbf{A}_i$, r_{ij} where $j \in \mathbf{S}_i$.
Ensure: $\ell \leq 100$ **or** $\|\mathbf{p}_i^\ell - \mathbf{p}_i^{\ell+1}\| < \varepsilon$.

Initialize: $\ell \leftarrow 0$
repeat
  Wireless receptions: $\mathbf{p}_j^\ell \leftarrow j \in \mathbf{S}_i$
  $\delta_i^\ell \leftarrow \frac{\mathrm{F}(\mathbf{p}_i^\ell)}{N_i}$
  $\min\limits_{\mathbf{p}_i^{\ell+1}} \left(\sum\limits_{j \in \mathbf{S}_i} \alpha_{ij}^\ell(\mathbf{p}_i^{\ell+1}) + \sum\limits_{k \in \mathbf{A}_i} \beta_{ik}^\ell(\mathbf{p}_i^{\ell+1}) \right)$
  subject to :
  $p_{x_i}^\ell - \delta_i^\ell \leq p_{x_i}^{\ell+1} \leq p_{x_i}^\ell + \delta_i^\ell$
  $p_{y_i}^\ell - \delta_i^\ell \leq p_{y_i}^{\ell+1} \leq p_{y_i}^\ell + \delta_i^\ell$
  $\mathbf{p}_i^{\ell+1} = \frac{\mathbf{p}_i^{\ell+1} + \mathbf{p}_i^\ell}{2}$
  Broadcast: $\mathbf{p}_i^{\ell+1}$
  $\ell \leftarrow \ell + 1$
until $\ell \leq 100$ **or** $\|\mathbf{p}_i^\ell - \mathbf{p}_i^{\ell+1}\| < \varepsilon$

Finally, as part of the collaborative process, all sensors broadcast their position updates in order to start a new iteration. In order to limit the amount of iterations (i.e., wireless transmissions), our algorithm uses

$$\|\mathbf{p}_i^{\ell+1} - \mathbf{p}_i^\ell\| \leq \varepsilon \tag{17}$$

as a simple stopping condition. If the position updates of sensor s_i satisfy the stopping criterion (17) after J_i iterations, the sensor will transmit its final estimate $\mathbf{p}_i^{J_i}$ with a "*stopping flag*" indicating that it will no longer transmit position updates. Hence, neighboring sensors should use $\mathbf{p}_i^{J_i}$ on further position updates. In this way, all sensors will gradually stop the process of updating/broadcasting their positions. Clearly, larger values of ε will require a lower number of iterations and at the cost of larger localization errors.

Algorithm 1 summarizes the proposed distributed spatially-constrained and localized (DSCL) algorithm for WSN localization. It is distributed in the sense that each sensor computes its own location updates, and localized since each sensor relies only on information from other sensors within a local neighborhood (i.e., those that have single-hop connectivity). The spatial constrain represents a collaboration strategy that allows all sensors to iteratively update their positions. As with any iterative scheme, set of initial locations, $\mathbf{p}_i^0$, is required. Each sensor can compute an initial position using an anchor-based localization scheme similar to the ones reported in [38]. Also, we assumed that each sensor s_i knows positions and range measurements to at least three non-collinear anchors [39] and the range estimates r_{ij} to its neighboring sensors ($s_j \in \mathbf{S}_i$). At the end of each iteration, each sensor transmits its position update $\mathbf{p}_i^{\ell+1}$ to its neighbors. It is also assumed that sensors are equipped with the communication protocols needed to share position updates among each other [40].

V. Assessing Performance of DSCL Algorithm

In WSNs, two parameters are commonly used to evaluate the efficiency of iterative localization algorithms: the accuracy performance and the number of iterations to reach final position estimates [41]. Both parameters are related to factors like the number and location of anchors, accuracy in range estimations, coverage range in sensors, initial positions and the iterative algorithm by itself. Next, we evaluate our proposed algorithm under different WSN scenarios considering the effects of these factors.

A. Localization With a Real Indoor WSN Benchmark

In this subsection, we evaluate the localization capabilities of DSCL using the set of real network measurements reported in [42]. This data set has been used in other works [16], [43], making it a good comparison reference. The data set presents a fully connected network of 44 sensors randomly deployed in an office environment within a 14 by 13 area. The data set consists of ToA and RSS measurements. ToA measurement errors are Gaussian with a standard deviation around 1.84 m. For RSS, a log-normal model was assumed with an estimated standard deviation of 3.92 dB. Also, this WSN scenario uses four anchors located intentionally in the corners with the goal of increasing their quality [44] and avoiding the collinear anchor problem (see [28], [43]).

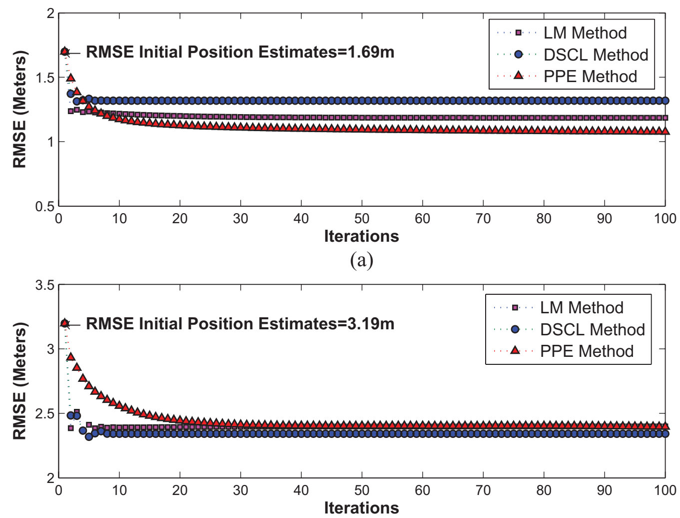
For comparison, we include localization results using the push-pull estimator (PPE) [22] and a distributed algorithm using the Levenberg-Marquardt (LM) algorithm [38], [45] to solve (11). The three algorithms were tested using similar procedures as in [16], [43]. A set of initial positions $\mathbf{L}^0$ was produced using the bilateration scheme reported in [38]. Each algorithm was ran for 100 iterations, and the accuracy of the localization process was assessed using root mean square error (RMSE)

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left\|\mathbf{p}_i^\ell - \mathbf{z}_i\right\|^2} \tag{18}$$

where $\mathbf{p}_i^\ell$ represents the final position estimate (with $\ell = 100$ iterations) for a sensor s_i. The evolution of the RMSE at each iteration is shown in Fig. 3(a) and (b) for ToA and RSS measurements respectively. In the case of RSS measurements, bias errors were removed as described in [16].

Fig. 3 shows all approaches converging to a minimum localization error at different rates. The RMSE curve for the PPE scheme has a smooth decay with convergence around 30 iterations for both ranging techniques. The LM and DSCL schemes present a fast decay towards a minimum point within 5 iterations. In the case of ToA, PPE reaches the lowest RMSE value followed by LM and DSCL. In our case, DSCL performance can be improved by increasing the resolution of the search region (e.g., 7×7 candidates) with a corresponding increase in computation. On the other hand, DSCL provides the best localization with the RSS measurements as depicted in Fig. 3(b).

A comparison of RMSE values after 100 iterations is shown in Table I. We include the RMSE for other well known localization schemes that have been tested with the same data set. As can be seen, DSCL localization performance is very competitive. In particular, it provides the second best RMSE for RSS measurements only outscored by the maximum likelihood estimation (MLE) scheme proposed in [43]. The latter is a centralized method requiring *a priori* knowledge of the distribution of range errors. Also, we should note that the dwMDS results reported in [16] were obtained by running dwMDS twice with a range threshold of six meters.



(a)

(b)

Fig. 3. Evolution of RMSE for ToA and RSS measurements as a function of the number of iterations for DSCL, LM, and PPE approaches. The three localization algorithms used the same set of initial location estimates.

TABLE I
RMSE AFTER 100 ITERATIONS OBTAINED WITH DIFFERENT LOCALIZATION ALGORITHMS

	Classical MDS	MLE	dwMDS	LM	PPE	DSCL
RSS	4.26 m	**2.18 m**	2.48 m	2.39 m	2.39 m	2.31 m
ToA	1.85 m	1.23 m	1.12 m	1.18 m	**1.05 m**	1.31 m

B. Localization Performance for Large WSN Deployments

In this subsection, we use network simulation to evaluate and compare the performance of DSCL for large sensor deployments. In our case, the localization process is comprised of two stages. First, a set of initial locations needs to be estimated using only the location information provided by GPS-enabled anchors. We assume that there are four anchors deployed randomly over the 2-D region. We can use single-hop schemes where anchors can connect to communicate their location directly to all sensors. The bilateration method proposed in [38] can be used for this purpose, provided that anchors have a large enough radio range. Another possibility is to use multihop schemes [46], [47] at the expense of a larger exchange of messages between nodes. On the second stage, DSCL iteratively computes position updates until some stopping condition is satisfied. Achieving good localization performance with a small number of iterations is crucial, since they are directly related to wireless transmissions and receptions. We assume a deployment of 100 sensors over a 2-D 100 by 100 m area. Given the distributed nature of DSCL, the size of the WSN is only limited by the availability of anchors for initialization.

TABLE II
MEAN RMSE AND STANDARD DEVIATION FOR INITIAL ESTIMATES OF 20 NETWORKS

Initialization Algorithm	Mean and SD over 20 WSNs	
	Mean RMSE	SD
LS Multilateration	22.7 m	2.22 m
Bilateration	12.96 m	0.84 m
LM	12.54 m	0.69 m

In order to reduce the number of wireless transmissions, we decided to use a single-hop initialization algorithm. Hence, our WSN model assumes that anchors can communicate to all sensors inside a 100 by 100 area. On the other hand, sensors are assumed to have a short radio range γ; a value of 30 meters has been used extensively. To ameliorate the uncertainty on the range estimates r_{ij}, each sensor s_i adjust this value according to

$$r_{ij} = \gamma \quad \text{if } r_{ij} > \gamma. \tag{19}$$

A similar geometrical relation has been reported in [16]. Also, in our experiments, we assume that γ ideally remains constant along all directions.

This simulation framework was devised after considering advances in hardware and wireless technologies at the time of this writing. For instance, in the case of a ZigBee-based network [48], [49], the nominal range of existing devices is 100 meters (or lower) with a power consumption of 1 mW. This technology has been typically considered as the baseline for sensors. Since anchors are required to self-localize and to have a large radio range, they have a higher power requirement. In practice, this could be achieved by equipping anchors with a ZigBee-PRO transceiver [50], [51]. The ZigBee-PRO standard is interoperable with regular ZigBee radios with a range upwards of 1.5 Km and a transmission power of 60mW in outdoor scenarios.

The remaining work in this paper focuses on RSS measurements. Although reliable RSS measurements are difficult to obtain [52], [53], RSS is an inexpensive ranging technology which has become widely available in wireless devices. We simulate RSS measurements using the propagation model described in (1). The free space path-loss formula [16], [54]

$$P_R(d) = \frac{P_T G_T G_R \lambda^2}{L(4\pi d)^2} \tag{20}$$

is used to obtain $P_0(d_0)$ at a reference distance of $d_0 = 1$ meter. In our model we set $L = G_T = G_R = 1$ and the carrier frequency to 2.4 GHz (corresponding to Zigbee radios). Based on Zigbee and Zigbee-PRO specifications, we assume transmission powers of $P_T = 60$ mW (+18 dBm) and $P_T = 1$ mw (0 dBm) for anchors and nodes respectively. Thus, we use $P_0(d_0) = -40$ dBm for sensors and $P_0(d_0) = -22.2$ dBm for anchors. Also, we use a representative path-loss factor $\eta_p = 2.6$ and $\sigma_{\mathrm{SH}} = 6_{\mathrm{dB}}$ for outdoor scenarios [11]. Finally, noisy pair-wise simulated ranges r_{ij} and R_{ik} are obtained by averaging 10 range measurements as reported in [28].

For the simulations that follow, we consider 20 different WSNs where each one is composed of $N = 96$ sensors and

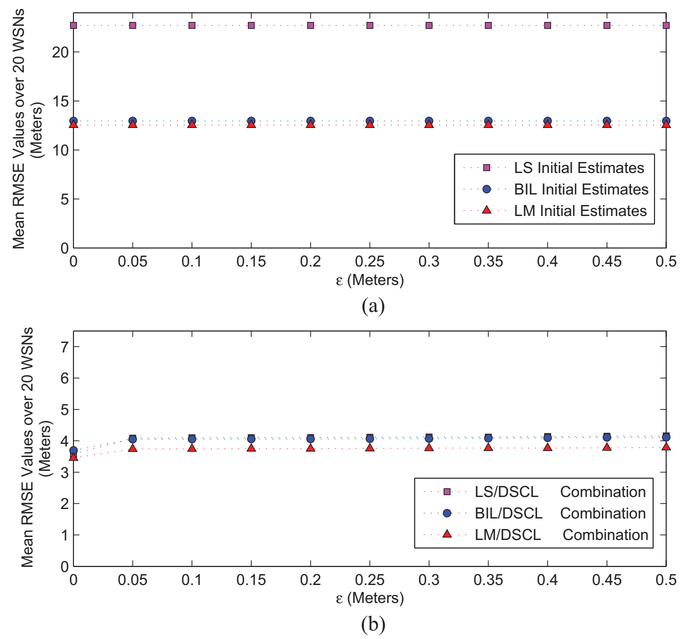


Fig. 4. (a) Least squares multilateration (LS), bilateration (BIL), and Levenberg–Marquardt (LM) algorithms providing initial estimates to iterative localization algorithms. (b) Accuracy versus stopping criteria of the DSCL iterative algorithm at different initial estimates.

$M = 4$ non-collinear anchors. Anchors and sensors are randomly distributed over a 100 by 100 m area. For each network, we generate a set of noisy ranges according to the Gaussian model in (3). Since the initial point for an iterative algorithm is closely related with its robustness and convergence, we also evaluate the impact of initialization schemes on the overall localization performance. Specifically, we evaluate the least-squares multilateration (LS), Levenberg-Marquardt (LM), and bilateration algorithms discussed in [38]. We should remark that the LM algorithm is being used for both initialization and iterative localization. The RMSEs of initial estimates over the 20 WSNs are summarized in Table II. For instance, the bilateration algorithm provides an average localization error (RMSE) of 12.96 meters with a standard deviation (SD) of 0.84 meters. Thus, given a set of initial estimates shown in Fig. 4(a) and a fixed threshold ε, each sensor s_i refines its position $\mathbf{p}_i^\ell$ using an iterative algorithm until either (17) is satisfied or the maximum number of iterations (bounded to 100) is completed.

We simulated the DSCL localization process for each one of the 20 networks over different values of $\varepsilon \in [0, 0.5]$. The particular case of $\varepsilon = 0$ refers to the situation where each sensor s_i will perform 100 iterations (i.e., 100 broadcasts of position updates). Fig. 4(b) presents the average RMSE versus. ε over the 20 WSNs. There are three combinations in the figure and their corresponding initial estimates, "■" presents RMSE values using the closed-form LS algorithm for initialization, "•" presents the bilateration results, and the "▲" curve presents the LM initialization algorithm.

As it was expected and confirmed in Fig. 4(b), incrementing the value of ε leads to slightly higher RMSEs since sensors stop their iterative process earlier. We observe that better position estimates are obtained using the LM initialization.

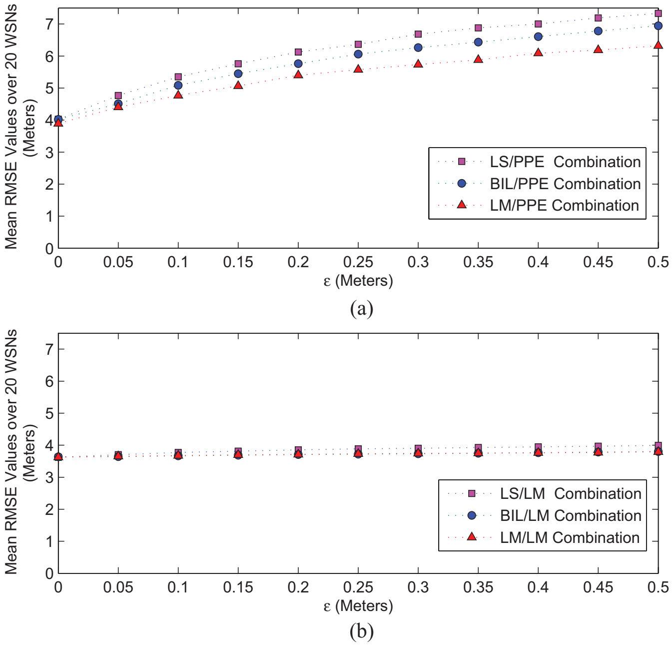


Fig. 5. Accuracy versus stopping criteria of (a) PPE and (b) LM iterative algorithms at different initial estimates provided by the least squares multilateration (LS), the bilateration (BIL), and the Levenberg–Marquardt (LM) schemes of Fig. 4(a).

Also, we observe that the performance slightly decreases when initial positions are generated with the LS multilateration or the bilateration algortihms.

Considering the same set of initial estimates, Fig. 5(a) shows results for the PPE algorithm. PPE shows higher RMSEs with respect to DSCL for all ε. For example, using the LM initialization algorithm with $\varepsilon = 0.5$ m and $\varepsilon = 0$ the DSCL approach achieved RMSE 3.78m and 3.45m respectively while the PPE provided 6.32 m and 3.89 m. In Fig. 5(b) we show the localization error performance of the iterative LM algorithm. Similar to DSCL, the localization error shows robustness with poor initial estimates and good performance for all values of ε. For example, while the LM/DSCL combination presents an error of 3.45 m for $\varepsilon = 0$, the LM/LM scheme presents an error of 3.62 m. Then, for $\varepsilon = 0.5$ m the LM/LM combination obtains an error of 3.79 m very close to the LM/DSCL scheme of 3.78 m. This indicates that our DSCL is very competitive when compared with LM, which is an improved gradient descent algorithm with a much higher computational complexity.

We summarize the results of this section in Fig. 6 where we plot the RMSE values versus ε for the three localization algorithms when the initial positions are estimated with the LM algorithm. The average localization error for LS increases by approximately 2.5 m (70%) for $\varepsilon = 0.5$ m. On the other hand, the RMSE for both DSCL and LM increases by less than .25 m for the same threshold. Moreover, their curves are almost flat for the full range of ε values. This behavior indicates that both algorithms achieve their final estimates within a few iterations implying a fast rate of convergence. The differentiating factor between the DSCL and LM algorithms is their computational complexity. DSCL solves local spatially-constrained local problems using a simple search over a set of candidate solutions. On the other hand, the LM algorithm has a very high computational cost (e.g., requires computation of gradients, Jacobians, and Hessians) within each iteration [38], [55]. In conclusion, DSCL is a suitable localization scheme that can be implemented and deployed on resource-constrained WSNs.

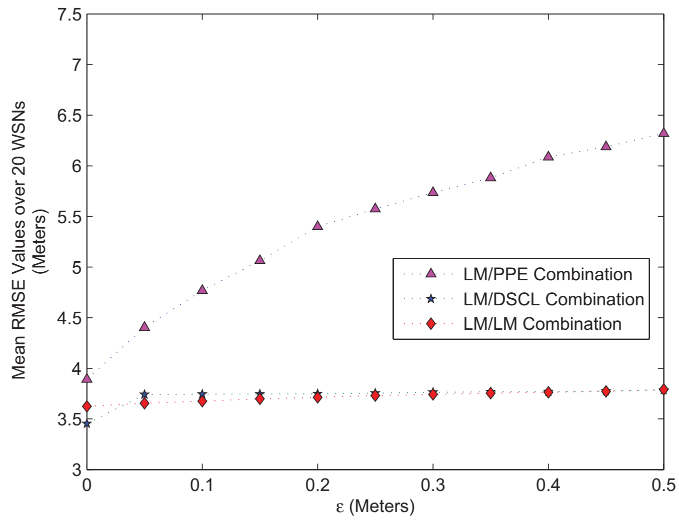


Fig. 6. Accuracy versus stopping criteria of three iterative algorithms considering LM initial position estimates.

C. Tradeoff Between Wireless Transmissions and Accuracy Performance

Iterative algorithms like the ones discussed in this paper require broadcasting their position updates at the end of each iteration. Wireless transmissions (and receptions) constitute the most energy expensive operations in a WSN. Hence, an important requirement is to design distributed algorithms that require a minimum exchange of messages between sensors. Next, we analyze the relationship between the number of wireless transmissions (i.e., iterations) and localization accuracy considering different threshold values in the stopping condition (17). We assume that all sensors use the same threshold value ε for a given simulation run. Let us define Ψ_{ij}^{ε} as the total number of wireless transmissions use by sensor s_i on the jth network to reach its final position estimate $\mathbf{p}_i^n$. Then, we define the total number of wireless transmissions needed by network j as

$$\varsigma_j^{\varepsilon} = \sum_{i=1}^{N} \Psi_{ij}^{\varepsilon} \tag{21}$$

where $N = 96$, $j = 1, \ldots, 20$, and ε can be varied between 0 and 0.5 m. Next, we define

$$\xi^{\varepsilon} = \frac{1}{20} \sum_{j=1}^{20} \varsigma_j^{\varepsilon} \tag{22}$$

as the average number of transmissions required by a WSN to complete the localization process for a given ε.

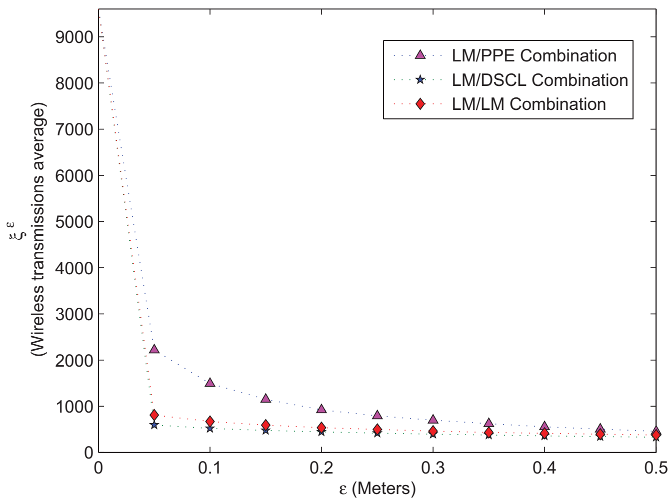


Fig. 7. Wireless transmissions average ξ^{ε} at each threshold ε using the LM initial estimates and three iterative algorithms PPE, DSCL, and LM.

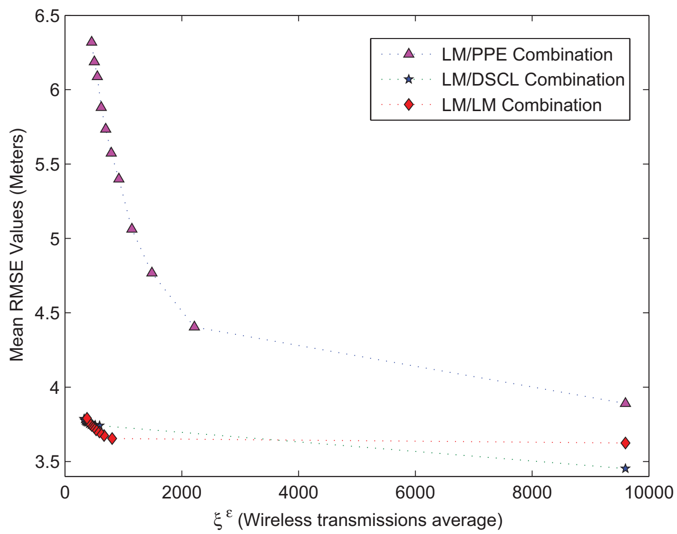


Fig. 8. Accuracy performance versus wireless transmissions for the iterative PPE, DSCL, and LM approaches at different thresholds with LM initial position estimates.

In Fig. 7, we plot ξ^{ε} for different threshold values. As expected, increasing ε decreases the number of wireless transmissions. The figure shows that small values of ε lead to a significant reduction of transmissions which leads to energy savings. Also, we note that for DSCL and LM, the number of transmissions remains flat as we increase ε.

As mentioned before, clearly, our approach is more efficient that the PPE scheme since it requires less number of transmissions to reach a certain RMSE value. On the other hand, similar to our algorithm the LM approach uses a small number of transmissions to reach good accuracy performance. Thus, LM and DSCL schemes will present similar results. For example, the LM/DSCL combination requires 330 wireless transmissions (on average) to reach a RMSE value of 3.78 m while the LM/LM combination requires around 384 wireless transmissions to reach the same RMSE value.

Fig. 8 summarizes the relationship between RMSE and wireless transmissions for the three iterative algorithms. We can see that small values of ε significantly reduce the number of transmissions while maintaining acceptable localization performance. We note that at $\varepsilon = 0$, the DSCL scheme achieves the lowest RMSE value with 9600 transmissions (96 sensors broadcasting their position updates 100 times). However, what is more relevant about Fig. 8 is the effect of the threshold over localization performance. For example, for $\varepsilon = 0,05$ m, the number of radio transmissions is reduced to less than 700 for the DSCL approach while maintaining practically the same RMSE values, implying that the proposed algorithm has a fast rate of convergence. On a similar way, the LM approach (almost overlapped with the DSCL scheme) has a fast rate of convergence providing an excellent accuracy performance at lower values in ε.

Finally, we should remark that even though wireless transmissions consume more energy than any other process in a sensor, other sources of energy consumption (i.e., wireless receptions, leakage energy, and CPU cycles [56]) must be considered for modeling the power consumption of sensor components. This analysis is beyond the scope of this paper. Detailed analysis regarding power consumption for wireless sensor networks can be found in [55], [57]. In particular, [55] presents a detailed model and power consumption analysis for localization algorithms.

VI. Conclusion

In this research, we have presented a distributed algorithm for range-based localization on wireless sensor networks. The method is an iterative and collaborative scheme where each sensor solves a series of local minimization problems. The resulting positions are broadcasted among neighbor sensors in order to repeat the next location update over the whole network. The algorithm performance has been tested using ToA and RSS measurements. The extensive evaluations over RSS measurements presented in this paper indicate that our algorithm is accurate and robust when compared with two well-known localization schemes. Also, the iterative algorithm shown to require a few iterations to reach accurate position estimates, implying savings on wireless transmissions in distributed localization schemes for WSNs. Besides, our algorithm has the property of being computationally efficient, and it can be implemented with the basic resources provided by the sensor.

We are interested on evaluating our distributed algorithm in combination with range-free [58]–[60] and multi-hop [1], [9], [18], [46], [61]–[63] algorithms in order to reduce the power requirement of anchors which have an impact on system cost. Finally, as part of our future efforts we plan to evaluate the performance of our algorithm under more realistic conditions including: signal fading, channel asymmetry, network latency, message routing, and sensor failure. Ultimately we aim at implementation, deployment, and testing of the proposed algorithm on a real wireless sensor network.

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