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Performance Evaluation of Classic and Accurate SVD Computation in a Multispectral Image Segmentation Problem

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Resumen: Matrices totalmente no-negativas (TNN) tienen un amplio rango de aplicaciones. Recientemente se introdujo un novedoso algoritmo para calcular la descomposición de valores singulares (SVD), el cual argumenta mejorar la precisión de las aplicaciones presentes. En este articulo, dicho algoritmo es explorado y probado en una aplicación de procesamiento de imágenes multiespectrales: detección de tormentas de arena (segmentación). Los datos multiespetrales son formulados como matrices TNN, y posteriormente se calculan la descomposición bidiagonal y los valores singulares para ser usados como vector de características. Cuando se compara el desempeño utilizando los valores singulares con el método tradicional y con el método de alta precisión, se encontró que este ultimo muestra un pequeño incremento sobre el método tradicional. Para una evaluación visual, también se presenta el evento de tormenta de arena en el centro de México el 18 de Marzo de 2008, lo cual confirma los resultados numéricos obtenidos.
Palabras Clave: tormentas de arena, procesamiento de imágenes multiespectrales, descomposición de valores singulares, redes neurales.

Abstract: Totally nonnegative (TNN) matrices have wide range applications. Recently a more accurate algorithm for computing singular value decomposition (SVD) was developed, promising to improve current application's precision. In this document, the algorithm is explored and tested in a multispectral image processing application: dust storm detection (segmentation). The multispectral data is posed as TNN matrices, then Bidiagonal Decompositions and Singular Values are computed for feature extraction. When we compared the traditional SVD numerical solution and the high relative accuracy SVD algorithm, we found that the latter shows slight improvement over the traditional approach. For visual assessment, we present the event of March 18, 2008, dust storm in central Mexico, and the visual results match with the numerical results.
Keywords: dust storms, multispectral image processing, singular value decomposition, neural networks.

Introduction

In pattern recognition, the feature vectors used to model the system are essential. From the observed features we can approximate a model or a pattern. Moreover, we can build statistical models from the features and extract the most essential information contained in the observed features. Similarly, singular values contain sufficient information to perform pattern recognition tasks [1]. Singular values are very sensitive, and a small change can lead to large variations in the data. Therefore, research is needed to find more accurate singular values estimation algorithms. Recently, Dr. Koev in [2,3] introduced a novel algorithm to compute singular values (SVD). He uses *Totally Nonnegative* (TNN) matrices and a bidiagonal decomposition to perform computations at a high relative accuracy. However, uncertainty remains on the true effects of high relative accurate singular values computation in pattern recognition applications. This research aims to study the true effects of using more accurate singular values in a real-life pattern recognition application. The information from this study will help to understand the accurate computation implications in the computational intelligence field.

In Section 2 we briefly explain the concept of TN matrices. The application to multispectral image classification is discussed in Section 3. In Section 4 we conclude this study.

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Totally Nonnegative Matrices

Matrices with nonnegative data minors are called TNN matrices. TNN's are useful in a variety of applications such as pattern recognition [4], feature extraction, and data mining [5,6]. In [2,3] the author presented new $O(n^3)$ algorithms for the computation of all eigenvalues and singular values of a nonsingular totally nonnegative matrix to high relative accuracy. Such algorithms were applied to multispectral image processing, and they will be described in further sections. Also, it is important to remark that since the uniqueness and existence of a bidiagonal decomposition is critical, the kind of matrices is limited to *contiguous submatrices of square nonsingular totally nonnegative matrices*. From here, we will denote as any $\mathbf{A}$nonsingular TN matrix.

Relevant Properties of TNN Matrices

The most relevant properties of TNN matrices are that the eigenvalues of any TNN are always positive and also real. The irreductible TNN matrices (also called *oscillatory*) and their eigenvalues are as distinct as their singular values. The product between two TNN matrices is also a TNN matrix. Also, a TNN is a bidiagonal matrix with positive diagonal and only one nonnegative off-diagonal entry. All TNN matrices are obtained by simple products of such bidiagonal matrices.

Bidiagonal Decomposition of TNN Matrices

The elementary principles of bidiagonal decomposition will be introduced here; however, a more detailed explanation can be found in [2,3,7].

It is widely known that small relative perturbations or changes in the elements of a TNN matrix can produce dramatic changes in the smallest eigenvalues, singular values and in $\mathbf{A}^{-1}$. Hence, the TNN matrix is said to be ill-constrained.

Following [2,3] we choose to represent a TNN as the product of non-negative bidiagonal matrices. Thus, the *Neville elimination* is used since this representation arises naturally in the process. It is well known that the Neville elimination reduces a matrix to its upper triangular form. Then, a zero value is inserted in the element $(m,1)$ by subtracting a multiple of the row $b_{m1}=\frac{a_{m1}}{a_{m-1,1}}m-1$ from the row m. Thus a zero is created at position $(m-1,1)$ by subtracting the multiple $b_{m-1,1}=\frac{a_{m-1,1}}{a_{m-2,1}}$ of the row $m-2$ from the row $m-1$, and so on. In that way, the total nonnegativity is preserved during the process. Therefore, all the multiples b_{ij} are nonnegative. This elucidates the decomposition

$$\mathbf{A}=\left(\prod_{k=1}^{m-1}\prod_{j=m-k+1}^{m}\mathbf{E}_j\left(b_{j,k+j-m}\right)\right)\mathbf{U},$$

where $\mathbf{U}$ is $m\times n$ upper triangular and

$$\mathbf{E}_j(x)=\begin{bmatrix}1&&&&\\&\ddots&&&\\&x&1&&\\&&&\ddots&\\&&&&1\end{bmatrix}$$

is $m\times m$ and differs from the identity only in the $(j,j-1)$ entry.

If we apply the same process to the transpose $\mathbf{A}^T$, it holds the following decomposition

$$\mathbf{A}=\left(\prod_{k=1}^{m-1}\prod_{j=m-k+1}^{m}\mathbf{E}_j\left(b_{j,k+j-m}\right)\right)\cdots$$
$$\mathbf{D}\left(\prod_{1}^{k=n-1}\prod_{n-k+1}^{j=n}\mathbf{E}_j^T\left(b_{k+j-n,j}\right)\right),\quad (1)$$

where $\mathbf{D}$ is an $m\times n$ diagonal matrix and $\mathbf{E}_j^T$ is $n\times n$. We are following the notation proposed in [2,3], where $\prod_{1}^{k=n-1}$ indicates that the product is taken for k from $n-1$ down to 1. So, the matrices

$$\mathbf{L}^{(k)}\equiv\prod_{j=m-k+1}^{m}\mathbf{E}_j\left(b_{j,k+j-m}\right)=\cdots$$
$$\begin{bmatrix}1&&&&&\\&\ddots&&&&\\&b_{k,m-k+1}&1&&&\\&&b_{k+1,m-k+2}&1&&\\&&&\ddots&\ddots&\\&&&&b_{m,m-1}&1\end{bmatrix}$$

and

$$\mathbf{U}^{(k)} \equiv \prod_{n-k+1}^{j=n} \mathbf{E}_j^T\left(b_{k+j-n,j}\right) = \cdots$$

$$\begin{bmatrix} 1 & & & & & & \\ & \ddots & b_{n-k+1,k} & & & & \\ & & 1 & b_{n-k+2,k+1} & & & \\ & & & 1 & \ddots & & \\ & & & & \ddots & b_{n-1,n} & \\ & & & & & 1 \end{bmatrix}$$

are $m \times m$ lower- and $n \times n$ upper-bidiagonal, respectively. Therefore, the total decomposition becomes

$$\mathbf{A} = \mathbf{L}^{(1)}\mathbf{L}^{(2)} \cdots \mathbf{L}^{(n-1)}\mathbf{D}\mathbf{U}^{(n-1)}\mathbf{U}^{(n-2)} \cdots \mathbf{U}^{(1)}.$$

The off-diagonal entries in $\mathbf{L}^{(k)}$ and $\mathbf{U}^{(k)}$ can be defined as $l_i^{(k)} \equiv \mathbf{L}_{i+1,i}^{(k)} = b_{i+1,k+i+1-m}$ and $u_i^{(k)} \equiv \mathbf{U}_{i,i+1}^{(k)} = b_{k+i+1-n,i+1}$.

The author in [1] uses either $l_i^{(k)}$ or $b_{i+1,k+i+1-m}$ to denote the nontrivial entries of $\mathbf{L}^{(k)}$ and so with $\mathbf{U}^{(k)}$. Therefore, we can say that the fundamental structure of the TNN matrices is given by the following Theorem [2,7].

Theorem 1. *A matrix $\mathbf{A}$ of the form $m \times n$ is TNN if and only if it can be uniquely factored as*

$$\mathbf{A} = \mathbf{L}^{(1)} \cdots \mathbf{L}^{(m-1)}\mathbf{D}\mathbf{U}^{(n-1)} \cdots \mathbf{U}^{(1)},$$

where $\mathbf{D}$ is an $m \times n$ diagonal matrix with its diagonal entries $d_i, i = 1, 2, \ldots, \min(n, m)$; $\mathbf{L}^{(k)}$ and $\mathbf{U}^{(k)}$ are lower and upper bidiagonal matrices, respectively, such that:

1. $d_i > 0$ for $i = 1, 2, \ldots, \min(m, n)$;
2. $l_i^{(k)} = 0, \quad i < m - k;\ \ u_i^{(k)} = 0, \quad i < n - k;$ *and* $l_i^{(k)} = u_i^{(k)} = 0,\ i > m + n - k$;
3. $l_i^{(k)} \geq 0, \quad m - k \leq i \leq m + n - k,$ *and* $u_i^{(k)} \geq 0,\ n - k \leq i \leq m + n - k$;
4. $l_i^{(k)} = 0$ *implies that* $l_{i+1}^{(k-1)} = 0$; $u_i^{(k)} = 0$ *implies* $u_{i+1}^{(k-1)} = 0$.

So, we denote the bidiagonal decomposition of a TNN matrix $\mathbf{A}$ as $\Psi(\mathbf{A})$. The (i, j)-th entry of $\Psi(\mathbf{A})$ is equivalent to the multiplier (b_{ij}), used to set the (i, j)-th entry in $\mathbf{A}$ to a zero value only when $i \neq j$, or the (i, j)-th entry of on the diagonal $\mathbf{D}$ when $i = j$. We can also conclude that the transpose of $\Psi(\mathbf{A})$ is $\Psi(\mathbf{A}^T)$ and also $\Psi(\mathbf{A}^T) = \Psi(\mathbf{A})^T$.

Example of Bidiagonal Decomposition of a TNN Matrix

For a more clear understanding, we provide an example of the totally non-negative matrix bidiagonal decomposition process.

As an example, we have the 3×3 TNN matrix $\mathbf{R}$, and we want to get $\Psi(\mathbf{R})$. So, we have

$$\mathbf{R} = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 13 & 69 \\ 28 & 131 & 852 \end{bmatrix} = \cdots$$

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & 7 & 1 \end{bmatrix}\begin{bmatrix} 1 & & \\ 4 & 1 & \\ & 8 & 1 \end{bmatrix}\begin{bmatrix} 1 & & \\ & 5 & \\ & & 9 \end{bmatrix}\cdots$$

$$\begin{bmatrix} 1 & 2 & \\ & 1 & 6 \\ & & 1 \end{bmatrix}\begin{bmatrix} 1 & & \\ & 1 & 3 \\ & & 1 \end{bmatrix},$$

and it is stored as

$$\Psi\left(\begin{bmatrix} 1 & 2 & 6 \\ 4 & 13 & 69 \\ 28 & 131 & 852 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Singular Values Computation and Complexity

The previously described decomposition method [2], is the key for an accurate and efficient computation of the singular values. First, given a TNN matrix the bidiagonal decomposition $\Psi(\mathbf{A})$ is computed. Second, Givens rotations are performed iteratively over $\Psi(\mathbf{A})$ until it is reduced to a bidiagonal matrix $\mathbf{F}$. Third, the singular values are computed using the traditional LAPACK method. Such algorithm can be computed in at most $O(\max(m^3, n^3))$ time.

The key for the success of this algorithm is that before using the LAPACK method, it does not perform any subtraction operations. In contrast, the traditional approaches perform several floating point subtraction operations leading to floating point error precision. Therefore, by the time LAPACK method is used, the original matrix $\mathbf{A}$ is posed in a form that minimizes floating point subtractions error.

In the following sections we address an application of the singular values based on bidiagonal decomposition, used as feature extraction method for multispectral image classification.

An Application to Image Segmentation

The segmentation of an image, $I(m,n) = \mathbf{X}$, can be described as a simple operation when $\mathbf{X} \in \{1, 2, \ldots, 10\}$; however, currently this is not the typical case. Instead we have $\mathbf{X} \in \{0, 1, \ldots, 255\}$ for an 8-bit digital image. In this section we describe an approach to multispectral image classification using accurate computations of $\mathbf{A} \subset T\{\mathbf{X}\}$ TNN matrices.

Contiguous Totally Nonnegative Nonsingluar Matrices

The most important restrictions of the algorithm for accurate TNN calculation is limited to matrices that are

1. Contiguous. $\mathbf{A}$:$a_{i,j} < a_{i,j+1} < a_{i+1,j} < a_{i+1,j+1}$. Ordered from minor to major across, row and column elements.
2. Totally Non-Negative. $\mathbf{A}$:$a_{i,j} > 0$. All the elements $a_{i,j}$ of $\mathbf{A}$ are non negative.
3. Non-Singular. $\mathbf{A} \neq \mathbf{A}^T$. The matrix $\mathbf{A}$ is different than $\mathbf{A}^T$.

The above constraints apply in Theorem 1 and must hold in order to utilize the algorithms and obtain relative high accuracy results.

From Images to TNN matrices

In this problem we addressed the particular case of a multispectral image $\mathbf{X} \in \Re^{n\times m}$ whose entries $x_{i,j}$ have nonnegative values $x_{i,j} > 0$. Then, let $\mathbf{x}^\alpha \in \Re^{p\times p}$ for $p = 9$, with entries $x^\alpha_{k,l} > 0$.

Such sub-matrices $\mathbf{x}^\alpha$ are constructed from $\mathbf{X}$'s entries $x_{i,j}$ as follows

$$\mathbf{x}^\alpha = \begin{bmatrix} x_{i-4,j-4} & \cdots & \cdots & \cdots & x_{i-4,j+4} \\ \vdots & \ddots & \vdots & & \vdots \\ \vdots & \cdots & x_{i,j} & \cdots & \vdots \\ \vdots & & \vdots & \ddots & \vdots \\ x_{i+4,j-4} & \cdots & \cdots & \cdots & x_{i+4,j+4} \end{bmatrix},$$

for all $i, j \in x_{i,j}$.

However, to make $\mathbf{x}^\alpha$ greater than zero and contiguous, we proposed to add a unitary constant $\mathbf{x}^\alpha + 1$ and then apply a row-column cumulative sum operation $\Lambda_\Sigma(\cdot)$, having that

$$\Lambda_\Sigma\left(\mathbf{x}^\alpha + 1\right) = \widehat{\mathbf{x}}^\alpha, \text{ or } \Lambda_\Sigma\left(x^\alpha_{k,l} + 1\right) = \widehat{x}^\alpha_{k,l}.$$

Note that the use of a function $\Lambda_\Sigma\left(\mathbf{x}^\alpha\right)$ in the sub-matrix does not results in loss of spatial information (assuming such a relationship exists between the elements of $x^\alpha_{k,l}$. Note also that this transformation is invertible: $\Lambda_\Sigma^{-1}\left(\widehat{\mathbf{x}}^\alpha\right) - 1 = \mathbf{x}^\alpha$.

Experimental Results in Image Analysis

The characteristic values or singular values of a matrix are very useful in pattern recognition [8,9], and a particular case is two dimensional data. A number of robust applications exist, and their solution is not trivial [10,11]. In spite of the fact that these applications use traditional Singular Values algorithms for their computations, they report good results. Therefore, we investigated whether for a simple pattern recognition problem, the Singular Values with high relative accuracy can produce better performance than using the traditional Singular Values algorithms.

Singular Values for Pattern Recognition

To produce Singular Values to high relative accuracy, we use the algorithm proposed in [2,3], which is based on the assumption of a bidiagonalization of a TNN matrix. Having a TNN matrix $\mathbf{x}^\alpha$, we can compute the singular values $\mathbf{y} = \Phi^{HRA}(\cdot)$ to a high relative accuracy with

$$\mathbf{y}^{HRA} = \Phi^{HRA}(\Psi(\widehat{\mathbf{x}}^\alpha)),$$

where $\mathbf{y}^{HRA}$ contains the singular values y^{HRA}_k to high relative accuracy, $\Phi^{HRA}(\cdot)$ is the function for the computation of the singular values to HRA, and $\Psi(\widehat{\mathbf{x}}^\alpha)$ is the bidiagonalization of the TNN matrix $\widehat{\mathbf{x}}^\alpha$. In the same way, we will denote the traditional singular values computation $\Phi(\cdot)$ as

$$\widetilde{\mathbf{y}} = \widetilde{\Phi}(\widehat{\mathbf{x}}^\alpha)$$

where $\widetilde{\mathbf{y}}$ contains the singular values of the TNN matrix $\widehat{\mathbf{x}}^\alpha$, and $\widetilde{\Phi}(\cdot)$ is the function for the traditional singular values computation algorithm.

For this particular problem we have designed a classification system based on the singular values of TNN matrices. This system uses a widely known neural network: a three layered feed forward back propagation neural network [12,13].

The neural network architecture described graphically in Fig. 1, was determined using the well known cross validation technique. The first (input) layer consists of 9 weighted inputs. The network has 5 neurons in the hidden layer with hyperbolic tangent sigmoid transfer function (*tansig*). It has a single neuron in the output layer with linear transfer function (*purelin*). The initial weights across the network are randomly generated, and all the 9 inputs correspond to the singular values. The network was trained with the Levenberg-Marquardt backpropagation algorithm (*trainlm*). The training was initialized with the default values: $\mu = 0.001$, maximum epochs=100, and goal=0. MATLAB and the Neural Networks Toolbox V6.0 were utilized in the design and experiments.

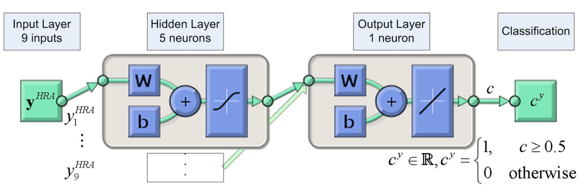

Fig. 1 Architecture of the three-layered feed-forward backpropagation artificial neural network.

After the composition of the architecture, we have designed a methodology to compare the results obtained when the neural network is trained, validated, and tested with and without high relative accuracy singular values of the TNN's extracted from images. Both neural networks are identically initialized. The methodology consists of classifying all the pixels of an image as part of a specific region of interest. In computer vision, this process is also known as image segmentation. We tested the impact of the accuracy of the singular values in an image segmentation task via neural networks. Hence, given the output $c \in \Re$ of the neural network, we have

$$c^y \in \Re = \left\{ \begin{array}{ll} 1, & c \geq 0.5 \\ 0 & \text{otherwise} \end{array} \right\}$$

where c^y is the final classification of the pixel, and1 denotes that the pixel belongs to the ROI, and 0 denotes that the pixel does not belongs to the ROI.

Comparison of Classification Numerical Results

We performed several experiments to compare the classification performance using classic and HRA. Numerical results are based on the following metrics:

$$\text{Precision}=\frac{\sum TP}{\sum TP+FP},$$

$$\text{Accuracy}=\frac{\sum TP+TN}{\sum TP+FN+FP+TN},$$

where TP stands for “True Positive,” FP “False Positive,” TN “True Negative,” and FN “False Negative.”

The experiments consists of training the neural network described in the previous section to distinguish a dust storm given multispectral imaging data. The features are the singular values of a 9×9 multispectral image sub-block. This sub-block is processed to make it a TNN matrix.

A database of approximately 750,000 feature vectors was used for our experiments. Only 375,000 were used for training the neural network, and the remaining were used for validation/testing purposes. We varied the number of training samples from 25,000 up to 375,000 in increments of 25,000.

The results of our experiments are summarized in Table 1, in which we show the accuracy and precision measures with and without HRA SVD computation. Also we show the difference between classification with classic SVD and HRA SVD. In Fig. 2 and 3 we present a plot of precision and accuracy performance metrics comparing the classic SVD and the HRA algorithm. It is clear that the HRA accuracy does not make a significant difference with fewer numbers of training samples. However, we can also conclude that in the presence of a large training data set, the performance increases slightly as shown in Fig. 4.

For a visual assessment of the classification results, we present in Fig. 5 the case of a dust storm in central Mexico on March 18, 2008. The result of dust storm classification with traditional SVD computation is shown in Fig. 6, while the result with the HRA SVD computation is shown in Fig. 7. Clearly, the visual results confirm the numerical results: HRA SVD perform a slightly better classification.

Table 1: Neural Network Classification Results with Classic SVD and with HRA SVD

Samples	Precision	Precision	Accuracy	Accuracy	Diff.
25000	0.3989	0.3989	0.5147	0.5147	0.0000
50000	0.4609	0.4610	0.5664	0.5665	0.0001
75000	0.4881	0.4883	0.5890	0.5892	0.0002
100000	0.5054	0.5058	0.6034	0.6038	0.0004
125000	0.5189	0.5195	0.6146	0.6152	0.0006
150000	0.5463	0.5471	0.6377	0.6385	0.0009
175000	0.5688	0.5700	0.6564	0.6575	0.0012
200000	0.5839	0.5854	0.6690	0.6705	0.0015
225000	0.5963	0.5982	0.6793	0.6811	0.0018
250000	0.6065	0.6087	0.6878	0.6900	0.0022
275000	0.6149	0.6174	0.6946	0.6972	0.0026
300000	0.6229	0.6259	0.7014	0.7043	0.0029
325000	0.6302	0.6336	0.7076	0.7110	0.0034
350000	0.6358	0.6396	0.7121	0.7160	0.0038
375000	0.6425	0.6468	0.7179	0.7222	0.0043

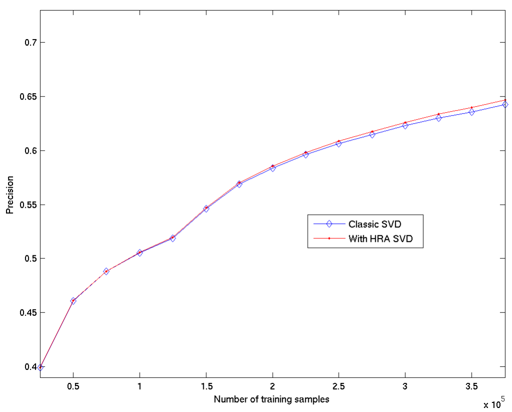

Fig. 2 Precision curve using a neural network with both classic SVD and with HRA SVD.

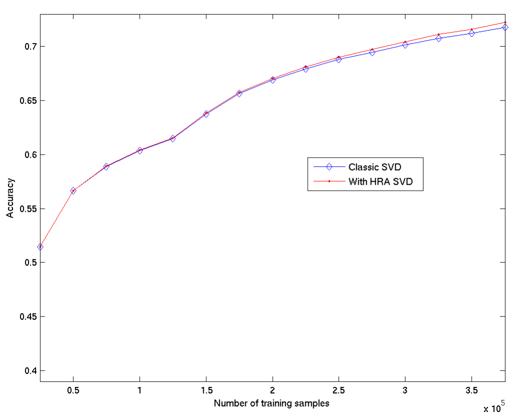

Fig. 3 Accuracy curve using a neural network with both classic SVD and with HRA SVD.

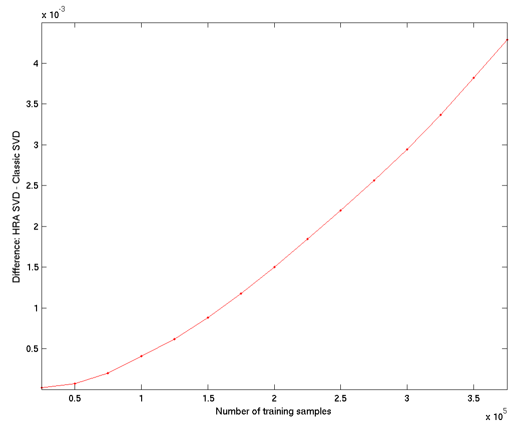

Fig. 4 Dierence between the accuracy curve with classic SVD and HRA SVD.

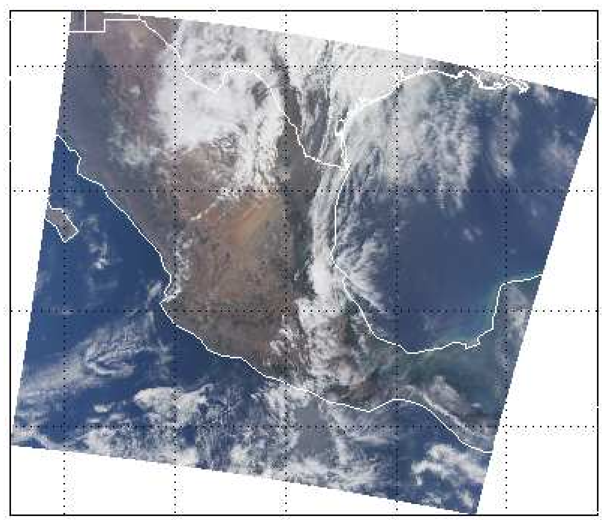

Fig. 5 True color image of the dust storm event at central Mexico on March 18, 2008.

Conclusions

We have studied the effects of high relative accuracy (HRA) singular value decomposition (SVD) on a simple pattern recognition task. We summarized the method proposed by Koev [2,3] for totally nonnegative matrices (TNN). Also we posed the problem of multispectral image classification to be suitable for HRA SVD computation. Then, we performed experiments with neural networks varying the number of training samples to observe the accuracy and precision obtained with and without HRA SVD.

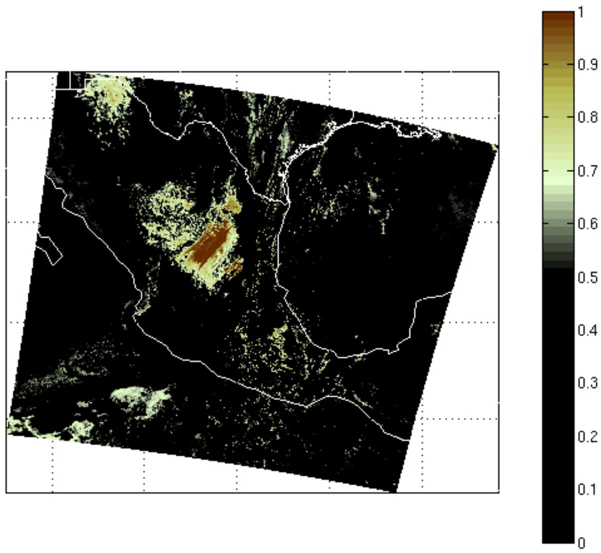

Fig. 6 Result of dust storm classification using traditional SVD computation.

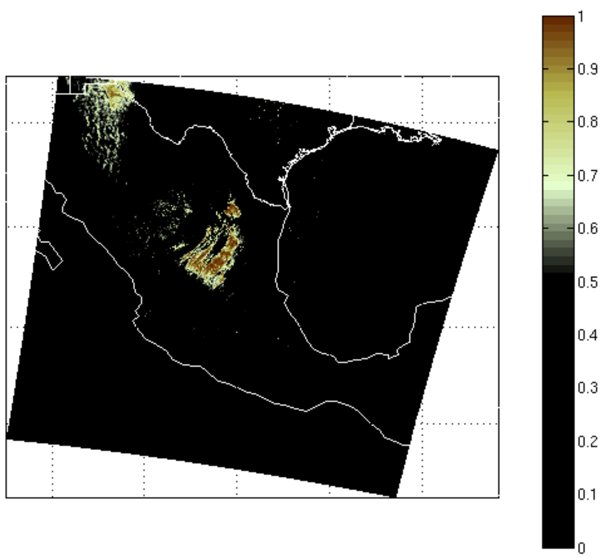

Fig. 7 Result of dust storm classication using High Relative Accuracy SVD computation.

Experimental results show that for smaller numbers of training samples, there is no significant difference in using HRA SVD or the traditional approach. However, when the number of training samples is large, the HRA SVD seem to improve classification performance and accuracy.

We have presented a visual example of multispectral image classification: dust storm detection. The visual assessment of the detection problem confirms the numerical results. The findings of this work are germane to the pattern recognition community. It is hoped that this information will lead to improved SVD-based pattern recognition applications that can be modeled as TNN matrices.

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